STUDY ON ABBE’S PRINCIPLE IN PARALLEL KINEMATICS

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1. INTRODUCTION

For a long time, conventional machine tools or coordinate measuring machines have employed the XYZ mechanism which is consisting of three mutually orthogonal slide mechanisms. It appears, however, the machine accuracy are already at their limit. One cause of the limitation is the violation of the Abbe's principle which is the basis of precision mechanism. In other words, end-effectors of the machines, namely the tool and the probe haven't been located in extensional lines of all scale units or guide ways. In short, there are some offsets between the scale unit axes and the measuring point or the machining point in the XYZ mechanism.

On the other hand, in the parallel kinematic, the tool and the probe can be located in the extension lines of the scale unit axes. Therefore, such the arrangement should decrease the influence on the mechanism motion error. In fact, through studying on coordinate measuring machine using parallel kinematics[1][2], the author could have found analytically and experimentally that the measurement error is minimized when the measuring point is located in all scale directions. In this paper, the effect of the joint motion errors on the mechanism motion error of Stewart platform mechanism has been analyzed by using a singular value decomposition. Moreover, the relation between the effect and various link layout has been discussed.

2. EFFECT OF ABBE'S OFFSET

Figure 1 shows one strut and a stage with the angular motion error caused by the motion errors of the joints, namely runouts of the joints. To keep the moving accuracy of the stage, it is necessary that the expansion and the contraction of the strut determine the variation of distance between the tool tip and the joint on the base. Figure (a) represents that the tool tip is not In the direction of the actuator and the scale. In other words, there is Abbe's offset between the tool tip and the strut. In this case, if the angular error motion of the stage occurs, the error on the longitudinal distance between the tool tip and the joint increases proportionally to the angular error and the offset.

On the other hand, figure (b) shows that the tool tip is In the direction of the actuator and the scale. In this case, being a secondary error, the error of the distance becomes negligible. Thus, it is expected that the influence of the joint motion error on the mechanism error provided the tool tip is in whole strut directions. However, such arrangement of the links should not be adopted because of a typical singular configuration of the Stewart platform.

3. METHOD OF ERROR ANALYSIS

As shown in figure 2, the motion errors of the spherical joints located on the base platform are decomposed into their components i.e. the radial direction $\delta\boldsymbol{r}_B=(\delta r_{B1},\ldots,\delta r_{B6})^T$, the circumferential direction $\delta\theta_B=(\delta\theta_{B1},\ldots,\delta\theta_{B6})^T$, and $\boldsymbol{Z}$ direction $\delta\boldsymbol{z}_B=(\delta z_{B1},\ldots,\delta z_{B6})^T$. In the same way, the motion errors of the joints on the stage are decomposed into $\delta\boldsymbol{r}_S$, $\delta\theta_S$, $\delta\boldsymbol{z}_S$ and $\delta\boldsymbol{l}_S$. For example, relationship between minute displacements $\delta\boldsymbol{r}_B$ and minute expansions and contractions of the six actuators $\delta\boldsymbol{l}=(\delta l_1,\ldots,\delta l_6)^T$, therefore, is shown as follows,

$$\delta\boldsymbol{l} = J_{rB}\delta\boldsymbol{r}_B \,, \tag{1}$$

where J_{rB} is a 6X6 Jacobian matrix derived from the inverse kinematics. Relationship between $\delta \boldsymbol{l}$ and the motion errors of the stage $\delta \boldsymbol{x}=(\delta x,\delta y,\delta z,\delta\alpha,\delta\beta,\delta\gamma)^T$, moreover, is as follows,

$$\delta \boldsymbol{x} = J\delta \boldsymbol{l}, \tag{2}$$

where J is a 6X6 Jacobian matrix which also can be derived from the inverse kinematics. Consequently, relationship between the motion errors of the joints $\delta \boldsymbol{r}_B$ and the motion errors of the stage $\delta \boldsymbol{x}$ is as follows,

$$\delta \boldsymbol{x} = JJ_{rB}\delta \boldsymbol{r}_B . \tag{3}$$

Therefore, the magnitude of the effect of the joints motion error $\delta \boldsymbol{r}_B$ on the motion error of the stage $\delta \boldsymbol{x}$ can be estimated by calculating the singular values of the matrix JJ_{rB}. The effects of other direction components of the joint errors are obtained by using matrices $JJ_{\theta B}$, JJ_{zB}, JJ_{rS}, $JJ_{\theta S}$ and JJ_{zS}. This paper deals with only translational components, namely upper half of the Jacobian matrix.

In calculation, the radial position of the joints on the base i.e. the radius of the base platform, is 100 mm. The circumferential positions of the joints on the base θ_B is $120n^\circ \pm 30^\circ$ (n=1,2,3). The stage radius and the height between the base and the stage, moreover, have been decided so that the machining point is located at the isotropic point at which the resolutions of ***XYZ*** directions are equal. According as the variation of the tool extension l_T, the maximum singular value has been calculated.

4. RESULTS OF ERROR ANALYSIS

When the circumferential position of the joints on the stage θ_S are $120n^\circ \pm 30^\circ$, the extension lines of six struts are intersected at a point on ***Z*** axis. In above case, the singular values can not be calculated because of the singular configuration. Thus, we have set the circumferential position θ_S to $120n^{\circ}\pm29.9^{\circ}$($n$=1,2,3). When a new parameter defined by the following equation is introduced, the angle difference θ is 0.1°.

$$\theta = |\theta_S - \theta_B| \tag{4}$$

Figure 3 shows the relation between the maximum singular value and the tool extension. In the figure, the singular values of any matrices have a minimum at the tool extension l_T of 22.62 mm. The tool tip is near whole extension lines of the struts then. In contrast, when the tool extension l_T is not 22.6mm, the singular values increase considerably. Consequently, the joint error has a strong effect on the mechanism motion error when the angle difference θ is extremely small.

Figure 4 shows that the results of the base joints have the same manner as that of the joints on the stage. In the figure 3, the minimum of the singular values Jr_S and Jz_S are 1.134 and 1.802 respectively, when the tool extension l_T is 22.62mm. The ratio of above values agrees with a tangent of the strut elevation angle ($\tan35.26^{\circ}$=0.7071) as shown in figure 5. The singular values concerning the circumferential motion error of the joints, moreover, are very small. Considering the above mentioned, it is assumed that the error component in strut direction has strongly effect on the motion error of the stage.

Next, the motion errors of the spherical joints are decomposed into the strut direction (δp_S and δp_B) and two directions perpendicular to the strut direction (δt_S,δt_B,δn_B and δn_B) as shown in figure 6. Moreover, similar calculation has been performed under larger angle difference θ. Figure 7 shows the singular values when the circumferential position of the stage joints θ_S is $120n^{\circ}\pm60^{\circ}$($n$=1,2,3) and when that of the base joints θ_B is $120n^{\circ}\pm50^{\circ}$($n$=1,2,3), namely the angle difference θ is 10°. The maximum singular value of the matrix JJp_S approaches 0.71, and is minimized when the tool extension l_T is 22mm. The effect of the joint motion error in the strut directions on the stage motion errors becomes minimal then. The result of the matrix JJp_B is in agreement with that of the matrix JJp_S because both the directions (δp_S, δp_B) are identical. Moreover, the maximum singular values of matrices JJt_S, JJt_B, JJn_S and JJn_B approach 0 when the tool extension l_T is 22mm. They are less than 0.0002 when the tool extension is 0-50mm. Thus, any joint motion errors perpendicular to the strut direction have little effect on the stage motion error.

Furthermore, we have got the same results as figure 7 when the calculation has been performed under θ_S=–60°-30°so that the angle difference θ is 10°. The same results have been obtained also when the angle difference is –10. Thus, the singular values depend on the angle difference shown in equation 4.

Figure 8 shows the relationship between the singular value of the matrix JJp and a distance d from the tool tip to the extension line of the strut, when the optimum tool extension l_T makes the singular value minimal under the angle difference θ of 0.5°-120°. The singular value is from 0.7 to 1.2 over a wide range of the angle difference θ when the tool extension l_T is optimum. Therefore, the motion error of the stage is minimized regardless of the fact that the tool tip is not in the strut directions or in the scale directions.

CONCLUSION

The effect of the joint motion error on the mechanism motion error of Stewart platform has been analyzed by using a singular value decomposition. The joint error in the strut direction strongly affects the translational motion error of the mechanism. Effect of the joint motion error perpendicular to the strut direction is negligible. However, it is unnecessary to locate the tool tip in strut directions. In conclusion, optimum arrangement of the joints and the links decreases the influence of the motion errors of each elements on the mechanism motion error.

References

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[2] T. Oiwa: New coordinate measuring machine featuring a parallel mechanism, Proc. of 1st Int. conf. EUSPEN, 2 (1999) 320-323.

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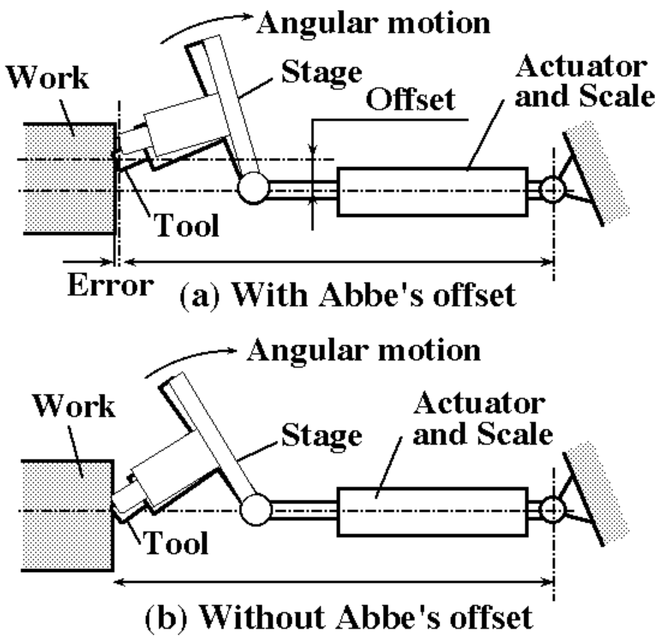

Figure 1 Influence of Abbe’s offset on motion error

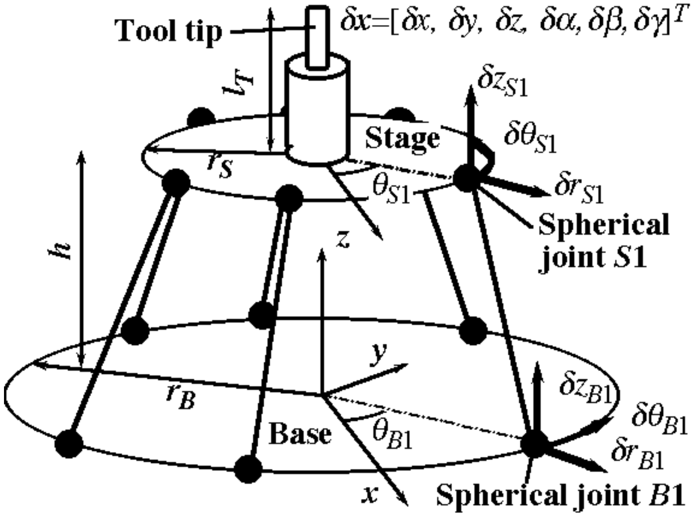

Figure 2 Runout of spherical joints on base and stage

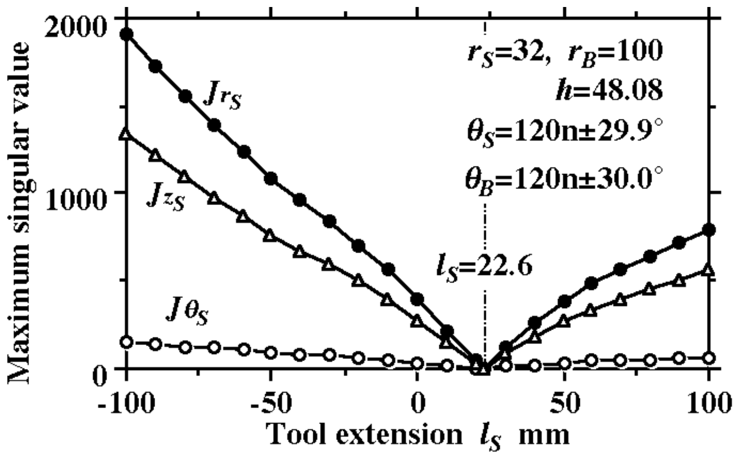

Figure 3 Relation between maximum singular value and tool extension (spherical joint on stage)

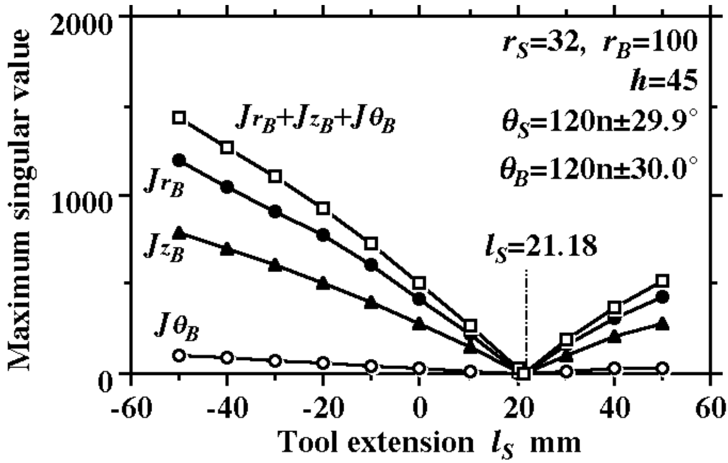

Figure 4 Relation between maximum singular value and tool extension (spherical joint on base)

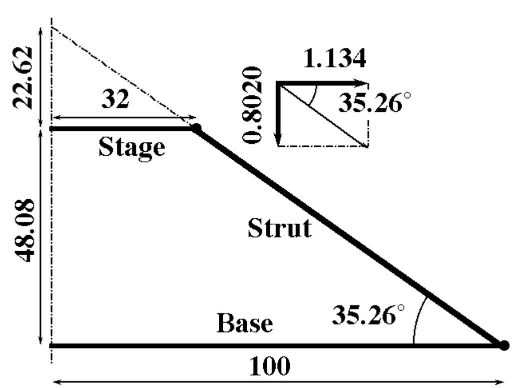

Figure 5 Link layout used in analysis and tangent of singular values

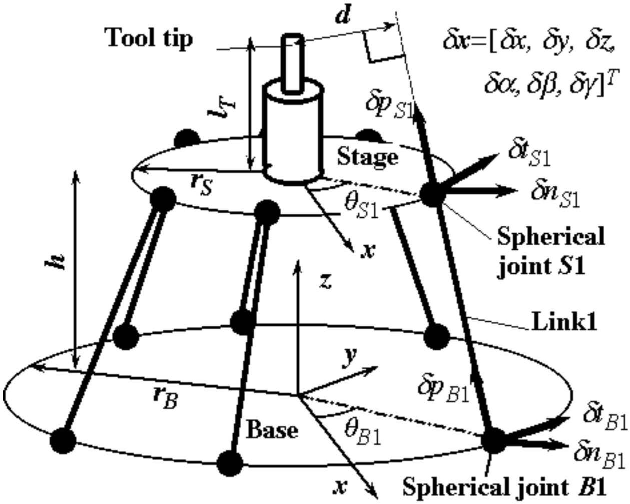

Figure 6 Joint runouts in strut direction and the other directions

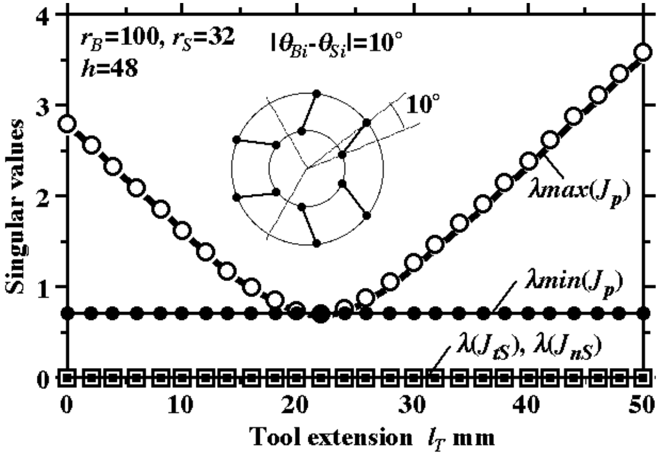

Figure 7 Relationship between tool extension and singular value

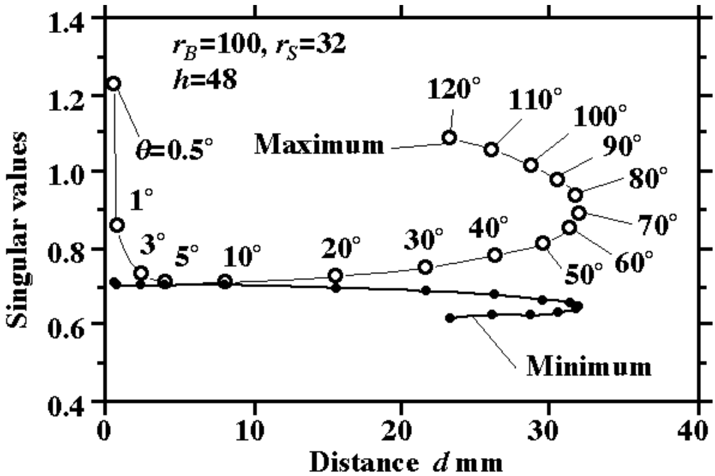

Figure 8 Relation between distance d and singular value in various link layouts