# **Accuracy Improvement of Parallel Kinematic Machine**-Error Compensation System for Joints, Links and Machine Frame-

#### Takaaki OIWA

Department of Mechanical Engineering, Faculty of Engineering, Shizuoka University Johoku 3-5-1, Hamamatsu 432-8561, JAPAN tmtooiw@ipc.shizuoka.ac.jp, http://oiwa.eng.shizuoka.ac.jp/

#### **Abstract**

This paper describes compensation methods for deformations of both the parallel kinematic mechanism and the frame supporting the mechanism. In general, these deformations are caused by the heat, the internal forces and the external forces during the processes. First, runouts and deformations of spherical joints and revolutionary joints are measured and compensated by nine displacement sensors built in above joints or by linear scale units built in the prismatic joints. Second, the longitudinal deformation of the prismatic joints is mechanically compensated by Super-Invar rod connected to the joints and the linear scale units. Moreover, nine displacement sensors with nine Super-Invar rods measure the variation of distances between the surface plate and three spherical joint supports. Consequently, the forward kinematics of Hexapod mechanism calculates the displacement and the attitude variations of the mechanism during operating from measured data. Last, the coordinates of the end effector are compensated by these displacements and attitude of the mechanism. Experimental results show reduction of the influence of the temperature fluctuation and variation of measured values.

#### 1. Introduction

When the improvement of the machining accuracy and the measurement accuracy is needed, it is extremely important to obtain an accurate relative position between the tool and the workpiece of the machine tool or the coordinate measuring machine. In actual machine, however, internal and external disturbances shown in figure 1 cause considerable relative positioning errors. Thus, not only moving accuracy but also structural and thermal stability in the whole machine is required. In general, much improvement for the guide element accuracy and the structural stiffness has been performed to decrease the motion error and the deformation caused by the external and internal forces. Increased mass raised along with it, however, further affects the motion error and the elastic deformation because of its own weight. To compensate the thermal deformation mentioned above, many researchers have generally investigated some prediction methods with limited temperature sensors. However, the thermal deformation is hard to predict because such temperature sensors can not measure the temperatures of

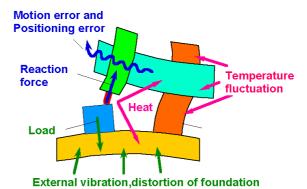


Fig. 1 Various causes of relative positioning error between tool and workpiece for conventional machine

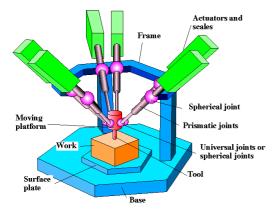
the whole elements of the machine. Moreover, the prediction in the machine comprised of beam structure, e.g. orthogonal slide mechanism, involves the great difficulties because of its complexity of the deformation at rapid fluctuation of the room temperature.

This study deals with improvement of the mechanism accuracy of parallel kinematic machine consisting of closed loop link mechanism. In the same manner as the orthogonal mechanism, the parallel kinematic machine has actually the following factors that cause the positioning error;

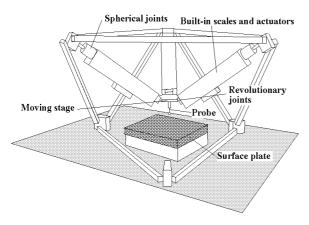
- (1) joint runouts caused by the motion of the mechanism,
- (2) elastic deformations of the links and the joints, generated by transfer of the center of gravity,
- (3) elastic deformation of the machine frame supporting the mechanism, which is caused by internal and external forces,
- (4) thermal expansion of the links and frame, which is caused by the temperature fluctuation.

Ota et al. [1] proposed a gravity compensation that considers elastic deformations due to the gravity the kinematic parameters. To improve the pose accuracy of Parallel kinematic machine, there are many works called kinematic calibration[2]. However, these are not suitable for unexpected disturbances and the temperature fluctuation because they deal with static or systematic kinematic errors.

This paper describes a compensation system consisting of some measuring devices for whole factors mentioned above. This system takes advantage of truss



(a) Hexapod type with six active prismatic joints



(b) Tripod type with three active prismatic joints[3][5]

Fig. 2 Two types of parallel mechanisms consisting of six and three active prismatic joints

structure in the parallel kinematic machine employing actuated prismatic joints with the linear scale units shown in **figure 2**.

## 2. Compensation for parallel kinematics 2.1 Joint errors

Recent studies[3]-[5] reported that the translational joint errors in a direction of actuated prismatic joint strongly affect the motion error of the mechanism. Effects of the joint motion errors except above direction, moreover, are minimized when the end effector is located on extensional lines of the prismatic joints. Therefore, variation of the prismatic joint expansion measured by the linear scale unit can be compensated by displacement sensors built in the spherical joint as shown in **figure 3**. Consequently, a length of *i*-th actuated link, *li*, can be expressed by

$$l_i = l_{0i} + g_{Li} - g_{Bi} - g_{Si} \tag{1}$$

where  $l_{oi}$  is initial length of the link,  $g_{Li}$  is measured value of the scale unit,  $g_{Bi}$  and  $g_{Si}$  are measured values of the displacement sensors installed in both spherical joints. The reference ball built in the joint must have proper sphericity to be a reference surface for the measurement. The reference balls can be installed into a universal joint shown in **figure 4**(a) and a revolutionary joint shown in figure 4(b).

#### 2.2 Compensation of link length

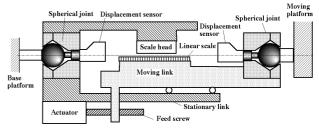
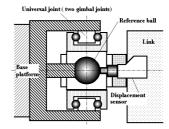
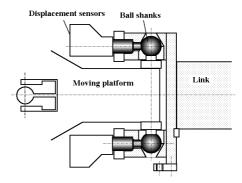


Fig. 3 Arrangement of two displacement sensors for measuring spherical joint motion error



(a) Reference ball and sensor in universal joint



(b) Two sensors in rotational joint using two ball shanks

Fig. 4 Installation examples of displacement sensors in universal joint and rotational joint

The fundamentals of compensation for the thermal expansion and the elastic deformation of the links are shown in figure 5. In general, some materials with low thermal expansivity, e.g. Super-Invar; expansivity: 0.3-0.8 ppm/K, are used to decrease the thermal expansion of equipments. It is not the best way to use these materials for whole structure because the materials are often heavy, expensive and bad machinability. Thus, a scale head and a scale of the linear scale unit are connected to both spherical joints through Super-Invar rods as shown in figure. Consequently, the distance between the scale unit and the spherical joints is not influenced by any thermal expansion of the link. The scale head and the scale are guided by linear bearings so that these may be moved only in the prismatic joint direction. Furthermore, the distance is not also influenced by any external force i.e. compressive or tensile forces, because no load is applied to the rods and the scale unit.

**Figure 6** shows an integrated compensation system for both the joint runout and the link expansion. Contacting a rod end with the spherical surface enables the scale unit to substitute for two displacement sensors. At this case, it is necessary to press the rod against the ball with the fixed contact force by using some springs.

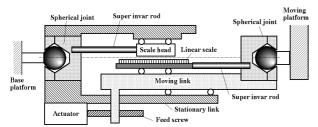
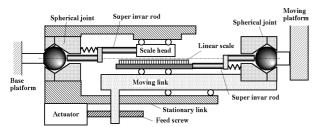


Fig. 5 Arrangement of two super invar rods for mechanical compensation of thermal deformation of link



**Fig. 6** Compensation device using super invar rods connecting with reference balls and scale unit

#### 3. Compensation for frame deformation

#### 3.1 Fundamentals

As mentioned in introduction, whole machine parts inclusive of the frame, base and mechanism are deformed by various causes. The base and the foundation on which the machine is set are not exception. Therefore, in this paper, the surface plate is employed as the reference surface. If the position and the attitude of the base platform in a fixed coordinate located on the surface plate are measured in process, the thermal and elastic deformations of the frame can be compensated. These compensation can be done independently of the structural deformations of the machine base and frame.

In general, three joints on the base platform of 3 DOF parallel kinematics are located at regular intervals of 120 degrees. Moreover, because two of six joints on the base platform of Hexapod are closely located, common parallel kinematic machines have three supports for base platform joints.

Then, when three reference points are set on the surface plate as shown in **figure 7**, its position and attitude can be computed by these reference points. Moreover, when three reference points are set on the base platform, its position and attitude also can be represented by these points. Consequently, the base platform position and attitude observed in the coordinate system located on the surface plate, <sub>M</sub>, are derived from six distances,  $u_1$ - $u_6$ , and three distances,  $t_1$ - $t_3$  as shown in figure 7. The above geometrical relationship forms an octahedron as shown in figure 8. Variations of those position and attitude caused by the thermal expansion and the elastic deformation, therefore, can be calculated by using the direct kinematics of Hexapod, if installed nine displacement sensors measure nine distances shown in figure 7.

### 3.2 Procedures for Calculation of position and attitude of mechanism

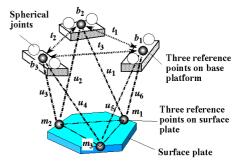


Fig. 7 Six reference points on surface plate and base platform

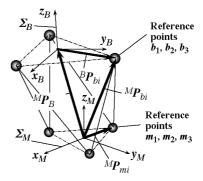


Fig. 8 Two coordinate systems including of reference points on base platform and surface plate

First, position vectors of the reference points,  ${}^{B}\boldsymbol{P}_{bi}$  (i=1, 2, 3), observed in the coordinate system fixed on the base platform,  $_{\rm B}$ , are calculated from three distances between the reference points on the base platform,  $t_1$ -  $t_3$ . Here, the origin of the coordinate system,  $_{\rm B}$ , is located at the center of gravity of the points  $b_i$ . The XY plane of  $_{\rm B}$ , moreover, contains three points  $b_i$ . In the same way, a coordinate system attached on the surface plate,  $_{\rm M}$ , is represented by three points  $m_i$ . Position vector,  ${}^{M}\boldsymbol{P}_{bi}$ , representing the reference points observed in the coordinate system  $_{\rm M}$  is

$${}^{M}\boldsymbol{P}_{bi} = {}^{M}\boldsymbol{P}_{B} + {}^{M}\boldsymbol{R}_{B}{}^{B}\boldsymbol{P}_{bi} , \qquad (2)$$

where  ${}^{M}P_{B}$  represents the position vector of the base platform and the transformation matrix  ${}^{M}R_{B}$  represents the attitude of the base platform.

Furthermore, the distances between the reference points of the base platform and the surface plate,  $u_1$ - $u_6$ , are

$$u_{1} = |^{M} \mathbf{P}_{b1} - {}^{M} \mathbf{P}_{m1}|, \quad u_{2} = |^{M} \mathbf{P}_{b1} - {}^{M} \mathbf{P}_{m3}|,$$

$$u_{3} = |^{M} \mathbf{P}_{b2} - {}^{M} \mathbf{P}_{m1}|, \quad u_{4} = |^{M} \mathbf{P}_{b2} - {}^{M} \mathbf{P}_{m2}|,$$

$$u_{5} = |^{M} \mathbf{P}_{b3} - {}^{M} \mathbf{P}_{m2}|, \quad u_{6} = |^{M} \mathbf{P}_{b3} - {}^{M} \mathbf{P}_{m3}|. \quad (3)$$

To obtain the position and attitude of the base platform from six distances  $u_1$ - $u_6$ , it is necessary to numerically solve nonlinear simultaneous equation. This equation is well known as the direct kinematics for Hexapod. Otherwise, infinitesimal variations of the position  ${}^{M}P_{B}$  and the attitude  ${}^{M}R_{B}$  of the base platform can be estimated by using the instantaneous kinematics because variations of the distances  $u_i$  are infinitesimal. Thus, the variations of the position and the attitude are expressed by

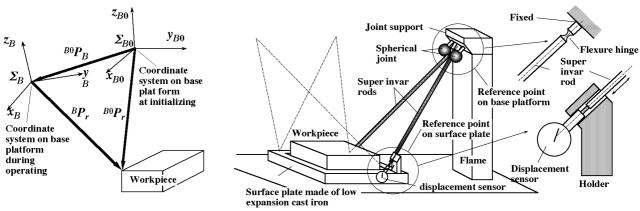


Fig.9 Two coordinate systems at initializing and during operating

Fig.10 Measurement device for frame deformation with 6 rods

$$U = J X, (4)$$

where  $U = \{u_1, ..., u_1\}^T$  represents the variation of the distance  $u_i$ ,  $X = \{x, y, z, roll, pitch, yaw\}^T$ represents the position and the attitude of the base platform. Jacobian matrix J is derived easily from the inverse kinematics of Hexapod. Thus, the inverse matrix of Jacobian  $J^{-1}$  leads the variation X as following equation;

$$X = J^{-1} \quad U. \tag{5}$$

In the manner described above, each three reference points are set on both platform and surface plate. Even if number of the reference points on the base platform is six, the calculation can be done successfully.

#### 3.3 Compensation for position of end-effector

In this chapter, the procedures will be given below to compensate the position and the attitude of the

- (1) An initial coordinate system of the base platform, BO is defined by using initial lengths of the distances of  $u_{0i}$  and  $t_{0i}$ , when the end effector returns to the machine zero point.
- (2) Variations of the distances,  $t_i$  and  $u_i$ , are measured by the displacement sensors during operation. The distances at the moment are expressed by

$$t_i = t_{0i} + t_i$$
,  $(i = 1, \dots, 3)$  (6)

$$t_i = t_{0i} + t_i$$
,  $(i = 1, \dots, 3)$  (6)  
 $u_i = u_{0i} + u_i$ .  $(i = 1, \dots, 6)$  (7)

- (3) A position and an attitude of the base platform,  ${}^{M}P_{B}$ and  ${}^{M}R_{B}$ , observed in the instantaneous coordinate system <sub>B</sub> can be computed either by the forward kinematics of Hexapod or by the instantaneous kinematics.
- (4) In machine tools, the tool objective position  ${}^{BO}P_r$ , in the initial coordinate system <sub>B0</sub> is transformed to that in the instantaneous coordinate system <sub>B</sub> by

$${}^{B}\boldsymbol{P}_{r} = {}^{B}\boldsymbol{P}_{B0} + {}^{B}\boldsymbol{R}_{B0} {}^{B0}\boldsymbol{P}_{r} . \tag{8}$$

The tool objective attitude in the initial coordinate system  $B_0$ , moreover, is transformed to that in the coordinate system  $_{\scriptscriptstyle B}$  by transformation matrix  ${}^{B}R_{B0}$ . Then, the tool tip of the parallel kinematic mechanism is driven to the objective position  ${}^{B}\mathbf{P}_{r}$  in the coordinate system  ${}_{B}$ .

(5) In coordinate measuring machines, first, a coordinate of the probe tip  ${}^{B}\mathbf{P}_{r}$  in the base platform coordinate system <sub>B</sub> is calculated by the forward kinematics from

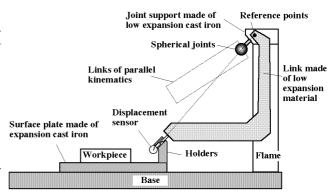


Fig.11 Measurement device for frame deformation with curved links

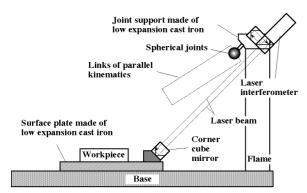
indicated values of the linear scale units. Second, the coordinate  ${}^{B}\mathbf{P}_{r}$  is transformed to a coordinate  ${}^{B0}\mathbf{P}_{r}$  in the initial coordinate system BO by

$${}^{B0}\boldsymbol{P}_{r} = {}^{B0}\boldsymbol{P}_{R} + {}^{B0}\boldsymbol{R}_{R} {}^{B}\boldsymbol{P}_{r} . \tag{9}$$

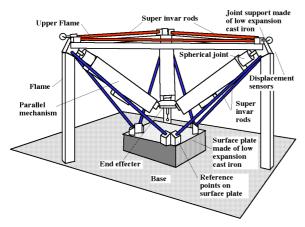
#### 3.4 Measuring device for frame deformation

Figure 10 shows an example of measuring devices using six Super-Invar rods for measuring the frame deformation. The spherical joints are mounted on joint support made of the low thermal expansion cast iron. The rods are spanned between the joint supports and the surface plate. One end of the rod is connected at a joint support through a flexure hinge. Another end, moreover, is supported by a holder, and can move to longitudinal direction. A displacement sensor is mounted on the holder, and can measures relative displacement of the end surface of the rod. The length of the rod is constant because the rod is not loaded by any external load. Moreover, the thermal expansion of the rod is infinitesimal at any temperature fluctuation. Therefore, measured displacement means the variation of the distance between the joint support and the surface plate. If the rods conflict against the work or the links of the mechanism, a curved profile of the rod as shown in figure 11 may be needed. If the distances between the surface plate and the joint supports are so long, some non-contact displacement sensors like a interferometer are available as shown in figure 12.

Figure 13 shows an example device for measuring the variation of the distance between each joint supports,  $t_i$ . Three Super-invar rods and three



**Fig.12** Measurement device for frame deformation using laser interferometer systems



**Fig.13** Measurement device for frame deformation in 3D coordinate measuring machine

displacement sensors are used in the same way as shown in figure 10.

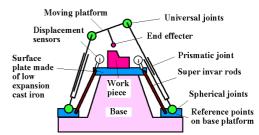
In the parallel kinematic machines described above, the mechanism is suspended from the frame. On the contrary, this compensation system is successfully performed, even if other configurations of the parallel kinematics are adopted, e.g. the base platform is located near the work side, as shown in **figure 14**, or the work is handling by the parallel kinematic machine, as shown in **figure 15**.

#### 4. Experiments

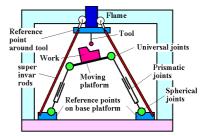
#### 4.1 Experiments in µCMM

Figure 16 shows an active link for a micro coordinate measuring machine. The link employs a compensation device shown in figure 6. The link is contracted and expanded by an active prismatic joint with a linear scale unit and inch worm mechanism using piezoelectric actuators. Moreover, variation of the link length is measured by Heidenhain linear scale unit, LIP-401R.

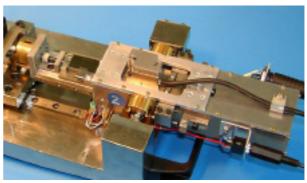
A test bed for the active link was made of low thermal expansion cast iron. The stage is connected to the link through a spherical joint, and is moved in longitudinal direction by the link. The displacement of the stage is measured by a laser interferometer. If there is no elastic deformation of the joint and the link when the external force is loaded, no difference will be observed between the displacements measured by both the scale unit and the interferometer. In actual, however,



**Fig.14** Arrangement of measurement device for frame deformation in another configuration



**Fig.15** Arrangement of measurement device for frame deformation in another configuration



**Fig.16** Active link consisting of prismatic joint with compensation device shown in figure 6

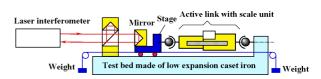
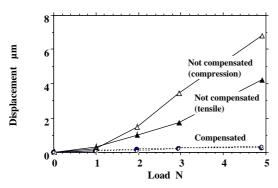


Fig.17 Experimental setup for active link subjected to external load

the difference is observed between them as shown in figure 17 when the compensation hasn't been made yet. However the compensation considerably decreases the displacement differences caused by the external force. Moreover, the compensation decreases the difference even when the link is expanded by temperature fluctuations. This result means that the scale unit can measure not only the displacement of the prismatic joint but also the deformation of the joints and the thermal expansion.

#### 4.2 Experiment in CMM

The compensation devices for the joint errors and the link expansion shown in figure 3 and 5 are installed in an experimental CMM[3][5] made by our laboratory as shown in **figure 19**. Nine electrical comparators are set into the joints to measure the



**Fig.18** Differences of measured displacements between scale unit and laser interferometer



**Fig.19** Experimental CMM with compensation device for frame deformation

displacements of the reference balls. Super-Invar rods, moreover, connect each joints and the scale unit to eliminate the influence of the thermal expansion. Moreover, the frame compensation device described in chapter 3 is also adopted for this machine. Other nine electrical comparators are mounted to measure the displacements of the frame as shown in figure 13.

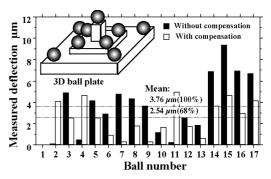
**Figure 20** shows deflections of measured distances between 17 balls mounted on a 3D ball plate. The compensation system decreases the deflections of 32% caused by the deformations of the joints.

Figure 21 shows variations of measured Z coordinate values of a reference ball on the surface plate. Measured values change gradually because of the temperature rise during measurement. The compensation decreases the deflection of the coordinate values. The changes in X and Y directions were so small because of an axisymmetric structure of the frame.

#### 5. Conclusions

Integrated compensation system consisting of some measurement devices has been proposed to improve moving accuracies of the parallel kinematic machine. The conclusions are drawn as follows.

- (1) The displacement sensors built in the joints and even the linear scale unit can measure the joint motion errors and the thermal expansion of the link.
- (2) Connecting the scale unit and the joints by Super-Invar rods compensates both the thermal expansion



**Fig.20** Measured deflection of distances between the balls of 3D ball plate

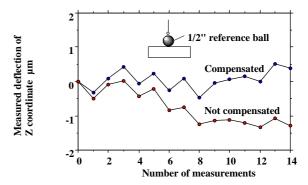


Fig.21 Variation of the measured coordinate during measurements

and the elastic deformation of the link.

(3) Measuring the changes of the distances between the surface plate and the joint supports can compensate the frame deformation caused by the temperature fluctuation.

#### Acknowledgments

The author would like to thank Mr. Oobayashi and Mr. Yamashita of Shizuoka University for their help in carrying out the measurements. This work is financially supported by the Grant-in-aid for scientific research of the Ministry of Education, Culture, Sports, Science and Technology.

#### References

- [1] H. Ota, et al., Study of Kinematic Calibration Method for Parallel Mechanism (3rd Report) -Gravity Compensation and Kinematic Calibration Considering Gravity-, J. Jpn. Soc. Prec. Egn., Vol. 67, No. 7, (2001), pp.1114-1119 (in Japanese).
- [2] E.g. C. W. Wampler, et al., An Implicit Loop Method for Kinematic Calibration and Its Application to Closed-Chain Mechanism, IEEE Trans. Rob. Autom., Vol. 11, No. 5, (1995), pp. 710-724.
- [2] T. Oiwa, et al., Coordinate Measuring Machine using Parallel Mechanism (3rd Report) -Abbe's Principle-, J. Jpn. Soc. Prec. Eng., Vol. 66, No. 9, (2000), pp. 1378-1382 (in Japanese).
- [3] T. Oiwa, et al., Study on ABBE's Principle in Parallel Kinematics, Proc. 2nd Chemnits Parallel Kinematic Seminar, (2000), pp. 354-352.
- [4] T. Oiwa, Coordinate Measuring Machine using Parallel Mechanism, Proc. 16th IMEKO World Congress, Vol. 8, (2000), pp. 211-214.